Wrapped branes with fluxes in 8d gauged supergravity

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ABSTRACT

We study the gravity dual of several wrapped D–brane configurations in presence of 4-form RR fluxes partially piercing the unwrapped directions. We present a systematic approach to obtain these solutions from those without fluxes. We use D=8 gauged supergravity as a starting point to build up these solutions. The configurations include (smeared) M2–branes at the tip of a G_2 cone on $S^3 \times S^3$, D2–D6 branes with the latter wrapping a special Lagrangian 3-cycle of the complex deformed conifold and an holomorphic sphere in its cotangent bundle T^*S^2, D3–branes at the tip of the generalized resolved conifold, and others obtained by means of T duality and KK reduction. We elaborate on the corresponding $\mathcal{N} = 1$ and $\mathcal{N} = 2$ field theories in $2 + 1$ dimensions.

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1 Introduction

The low energy dynamics of a collection of D–branes wrapping supersymmetric cycles is governed, when the size of the cycle is taken to zero, by a lower dimensional supersymmetric gauge theory with less than sixteen supercharges. The non-trivial geometry of the world–volume leads to a gauge theory in which supersymmetry is appropriately twisted [1], the amount of preserved supersymmetries having to do with the way in which the cycle is embedded in a higher dimensional space. In a generalization of the AdS/CFT correspondence [2], when the number of branes is taken to be large, the near horizon limit of the corresponding supergravity solutions provide gravity duals to the field theories arising on the world–volume of the D–branes [3]. The gravitational description of the strong coupling regime of these gauge theories allows for a geometrical approach to the study of such important aspects of their infrared dynamics as, for example, chiral symmetry breaking, gaugino condensation, confinement and the existence of a mass gap [3, 4].

An exhaustive study of the gravity/gauge theory correspondence for flat D–branes – whose low energy dynamics is dictated, in general, by non–conformal field theories– as well as the intricate phase structure of their RG flows, was undertaken in ref.[5]. In the case of theories with less than sixteen supercharges, the fact that there are too many possibilities in choosing the D–branes, the cycles, and the manifolds embedding them, prevent the very existence of an analogous comprehensive work. Yet, many cases have been considered so far [6]–[10]. A natural framework to perform the above mentioned twisting is given by lower dimensional gauged supergravities. Their solutions usually correspond to the near horizon limit of D–brane configurations thus giving directly the gravity dual description of the gauge theories living on their world–volumes. This approach, started in [3], has been widely followed throughout the literature on the subject.

Gauged supergravities have several forms coming from the dimensional reduction of the highest dimensional supergravities [11]. Turning them on amounts to the introduction of other branes into the system in the form of either localized or smeared intersections and overlappings. Many of these configurations correspond to extremely interesting supersymmetric gauge theories. In particular, these configurations give rise to a world–volume dynamics whose description, at different energy scales, is given by increasingly richer phases connected by RG flows. See, for example, [12]–[14].

The purpose of this article is to study the effect of turning on 4-form fluxes in the non compact directions of D6–branes wrapping supersymmetric cycles. We shall analyze different configurations that correspond to (smeared) M2–branes at the tip of a G_2 cone on $S^3 \times S^3$, D2–D6 branes with the latter wrapping a special Lagrangian 3-cycle of the complex deformed conifold and an holomorphic sphere in its cotangent bundle T^*S^2, D3–branes at the tip of the generalized resolved conifold, find their supergravity duals and explore their T duals. From the eleventh dimensional point of view, they amount to deformations of the purely gravitational backgrounds found in [7] corresponding to the small resolution of the conifold and a manifold of G_2 holonomy which is topologically $\mathbb{R}^4 \times S^3$. We shall construct these supergravity solutions by implementing the required topological twisting in maximal eight dimensional gauged supergravity [15]. We further elaborate on the corresponding $\mathcal{N} = 1$ and $\mathcal{N} = 2$ field theories in $2 + 1$ dimensions.

The plan for the rest of the paper is as follows. In section 2 we treat the case of D6–branes wrapping a special Lagrangian submanifold of the complex deformed conifold T^*S^3 in presence of 4-form fluxes. We derive the BPS equations both through the vanishing condition for supersymmetry transformations of the fermions as well as from the domain wall equations resulting from the effective Routhian obtained by inserting the ansatz into the 8d gauged supergravity Lagrangian. We obtain the general solution and uplift it to eleven dimensions. It describes a configuration of smeared M2–branes transverse to a (resolved) G_2 cone on $S^3 \times S^3$. We then discuss all possible reductions to D=10, their T duals, and the corresponding $\mathcal{N} = 1$ dual field theories in $2 + 1$ dimensions. In section 3 we include 4-form fluxes in a configuration of D6 branes wrapping a holomorphic S^2 in the cotangent bundle T^*S^2. We proceed following the same avenue of the previous section, and find a configuration of smeared M2–branes transverse to the generalized resolved conifold. Again, we study the reductions to D=10, their T duals, and the $\mathcal{N} = 2$ dual field theories arising in $2 + 1$ dimensions.

We discuss the results and present our conclusions in section 4. In appendix A we have collected the lagrangian and equations of motion of D=8 gauged supergravity, together with the corresponding uplifting formulae for the metric and the forms. We explain, in appendix B, how to find an effective lagrangian for a given ansatz for the eight dimensional fields when the 4-form G is turned on. This involves a subtle sign flip when constructing the Routhian after integrating out the corresponding potential. Finally, we show in appendix C that the deformation of the background produced by the inclusion of a 4-form amounts to the appearance of warp factors. This result generalizes a similar one recently obtained by Hernández and Sfetsos in the study of branes with fluxes wrapping spheres [16].

2 D6-branes wrapped on a 3-sphere with 4-form

In this section we will obtain supergravity solutions which correspond to D6-branes wrapped on a three sphere with a flux turned on its worldvolume. We will derive these solutions first in eight-dimensional gauged supergravity and then we will uplift the result to eleven and ten dimensions. The bosonic truncation of eight dimensional supergravity relevant for our purposes contains the metric $g_{\mu\nu}$, some scalar fields, an $SU(2)$ gauge potential A^i and a three-form potential (whose field strength we will denote by G). Notice that, in general, this is, an inconsistent truncation of Salam and Sezgin's theory: G acts as a non–linear source for some of the forms we have turned off. However, we will consider solutions that are fully compatible with the equations of motion of D=8 gauged supergravity. To this end, G must obey the following constraints:

$$G \wedge G = *G \wedge F^i = 0\,, \tag{2.1}$$

where F^i is the $SU(2)$ field strength and $*G$ is the Hodge dual of G in eight dimensions [1].

The D6-brane configurations we will be dealing with have some flux of G along the directions of the worldvolume which are not wrapped. We will first obtain them by looking

[1]For a similar discussion in $SO(4)$ seven dimensional gauged supergravity, see [9].

at the supersymmetric transformations of the fermionic fields which, after implementing the topological twisting and imposing some projection conditions on the supersymmetric parameter, give rise to some first-order differential equations for the metric and scalar fields. We will obtain the same first-order equations by looking at the effective lagrangian for our ansatz and verifying that its potential can be derived from a superpotential, whose associated *domain wall* equations are precisely those obtained from supersymmetry. Moreover, we will be able to find the general solution to these equations and the corresponding supergravity backgrounds in D=11 will be completely determined. We end this section by considering the KK reduction of our solution and several of its duals.

2.1 First-order equations from SUSY

Let us consider a D6–brane wrapping a special Lagrangian 3–cycle of the deformed conifold T^*S^3 from the point of view of eight-dimensional gauged supergravity. We will switch a 4-form G flux along three of the unwrapped directions of the brane and the radial direction. The presence of this flux introduces a distinction between one of the unwrapped directions of the brane and the other three. Accordingly, the ansatz for the metric will be:

$$ds_8^2 \,=\, e^{2f}\, dx_{1,2}^2 \,+\, e^{2\alpha}\, dy^2 \,+\, e^{2h}\, d\Omega_3^2 \,+\, dr^2 \;, \tag{2.2}$$

where $d\Omega_3^2$ is the metric of the unit S^3, f, α and h are functions of the radial coordinate r to be determined and $dx_{1,2}^2 = -(dx^0)^2 + (dx^1)^2 + (dx^2)^2$. The corresponding ansatz for the 4-form G in flat coordinates is:

$$G_{\underline{x^0 x^1 x^2 r}} \,=\, \Lambda\, e^{-\alpha-3h-2\phi} \;, \tag{2.3}$$

with Λ being a constant and ϕ the eight-dimensional dilaton. This ansatz for G ensures that its equation of motion is satisfied. Also, the first equation in (2.1) is satisfied. Moreover, we will parametrize the S^3 by means of the left invariant 1-forms w^i on the $SU(2)$ group manifold satisfying:

$$dw^i \,=\, \frac{1}{2}\,\epsilon_{ijk}\, w^j \,\wedge\, w^k \;. \tag{2.4}$$

In terms of three Euler angles θ, ϕ and ψ, the w^i's are:

$$\begin{aligned} w^1 &= \cos\phi\, d\theta \,+\, \sin\theta \sin\phi\, d\psi \;, \\ w^2 &= \sin\phi\, d\theta \,-\, \sin\theta\cos\phi\, d\psi \;, \\ w^3 &= d\phi \,+\, \cos\theta\, d\psi \;, \end{aligned} \tag{2.5}$$

while the metric of the unit 3-sphere is:

$$d\Omega_3^2 \,=\, \frac{1}{4}\sum_{i=1}^{3}\,(\,w^i\,)^2 \;. \tag{2.6}$$

A bosonic configuration of fields is supersymmetric iff the supersymmetry variation of the fermionic fields, evaluated on the configuration, vanishes. In our case the fermionic fields are two pseudo Majorana spinors ψ_λ and χ_i and their supersymmetry transformations are:

$$
\begin{aligned}
\delta\psi_\lambda &= D_\lambda \epsilon + \frac{1}{24} e^\phi F^i_{\mu\nu} \Gamma_i (\Gamma^{\mu\nu}_\lambda - 10\,\delta^\mu_\lambda \Gamma^\nu)\epsilon - \frac{g}{288} e^{-\phi}\epsilon_{ijk}\Gamma^{ijk}\Gamma_\lambda T\epsilon - \\
&\quad -\frac{1}{96} e^\phi G_{\mu\nu\rho\sigma} (\Gamma^{\mu\nu\rho\sigma}_\lambda - 4\delta^\mu_\lambda \Gamma^{\nu\rho\sigma})\epsilon\,, \\
\delta\chi_i &= \frac{1}{2}(P_{\mu ij} + \frac{2}{3}\delta_{ij}\partial_\mu\phi)\Gamma^j\Gamma^\mu\epsilon - \frac{1}{4} e^\phi F_{\mu\nu i}\Gamma^{\mu\nu}\epsilon - \frac{g}{8} e^{-\phi}(T_{ij} - \frac{1}{2}\delta_{ij} T)\epsilon^{jkl}\Gamma_{kl}\epsilon - \\
&\quad -\frac{1}{144} e^\phi G_{\mu\nu\rho\sigma}\Gamma_i \Gamma^{\mu\nu\rho\sigma}\epsilon\,. \qquad (2.7)
\end{aligned}
$$

In eqs.(2.7) the Γ's are 32×32 Dirac matrices and T^{ij} parametrizes the potential energy of the $SL(3,\mathbb{R})/SO(3)$ scalars of the theory, L^i_α, with $T = \delta_{ij}T^{ij}$ (see appendix A). The covariant derivative is:

$$
D\epsilon = (\partial + \frac{1}{4}\omega^{ab}\Gamma_{ab} + \frac{1}{4} Q_{ij}\Gamma^{ij})\epsilon\,, \qquad (2.8)
$$

where ω^{ab} are the components of the spin connection and Q_{ij} is an antisymmetric matrix constructed from the scalars and the $SU(2)$ gauge potential (see appendix A).

Following ref.[7], we will adopt an ansatz for the gauge field which corresponds to a complete identification of the spin connection with the R-symmetry gauge potential of the theory, namely:

$$
A^i = -\frac{1}{2g} w^i\,. \qquad (2.9)
$$

The corresponding field strength is $F^i = -\frac{1}{8g}\epsilon^{ijk} w^j \wedge w^k$. In this case it is possible to get rid of the coset scalars, L^i_α; T_{ij} and Q_{ij} simply reducing to $T_{ij} = \delta_{ij}$ and $Q_{ij} = -g\epsilon_{ijk} A^k$. For convenience, we will use the following representation of the Clifford algebra

$$
\Gamma^{\underline{a}} = \gamma^{\underline{a}} \otimes \mathbb{I}\,, \qquad \Gamma^{\underline{i}} = \gamma_9 \otimes \sigma^i\,, \qquad (2.10)
$$

where $\gamma^{\underline{a}}$ are eight dimensional Dirac matrices, σ^i are Pauli matrices and $\gamma_9 = i\gamma^{\underline{0}}\gamma^{\underline{1}}\cdots\gamma^{\underline{7}}$ ($\gamma_9^2 = 1$). The first-order equations we are looking for are obtained by requiring the vanishing of the right-hand side of eqs.(2.7). This is achieved after imposing some projection conditions on the supersymmetry parameter ϵ, which reduce the amount of supersymmetry to some fraction of that of the vacuum. The standard projections corresponding to the D6–branes wrapping the S^3 [7]:

$$
(\gamma_{\underline{r}} \otimes \mathbb{I})\epsilon = -i(\gamma_9 \otimes \mathbb{I})\epsilon\,, \qquad (\gamma_{\underline{ab}} \otimes \mathbb{I})\epsilon = -(\mathbb{I} \otimes \sigma^{ab})\epsilon\,, \qquad (2.11)
$$

have to be supplemented by a new one due to the presence of the G flux:

$$
(\gamma_{\underline{x^0x^1x^2}} \otimes \mathbb{I})\epsilon = -\epsilon\,. \qquad (2.12)
$$

The number of supercharges unbroken by this configuration is then one half of those corresponding to the case $\Lambda = 0$, *i.e.* two. It is now straightforward to get the following BPS equations:

$$
\begin{aligned}
f' &= -\frac{1}{2g}\, e^{\phi-2h} + \frac{g}{8}\, e^{-\phi} + \frac{\Lambda}{2}\, e^{-\phi-3h-\alpha} \,, \\
\alpha' &= -\frac{1}{2g}\, e^{\phi-2h} + \frac{g}{8}\, e^{-\phi} - \frac{\Lambda}{2}\, e^{-\phi-3h-\alpha} \,, \\
h' &= \frac{3}{2g}\, e^{\phi-2h} + \frac{g}{8}\, e^{-\phi} - \frac{\Lambda}{2}\, e^{-\phi-3h-\alpha} \,, \\
\phi' &= -\frac{3}{2g}\, e^{\phi-2h} + \frac{3g}{8}\, e^{-\phi} - \frac{\Lambda}{2}\, e^{-\phi-3h-\alpha} \,.
\end{aligned}
\qquad (2.13)
$$

Notice that, as it should be, eqs.(2.13) reduce to the first-order equations found in ref.[7] when $\Lambda = 0$.

2.2 First-order equations from a superpotential

Before finding the general integral of the BPS equations (2.13), let us derive them again by means of an alternative method which consists in finding a superpotential for the effective lagrangian constructed out of the introduction of our ansatz into the Lagrangian of D=8 gauged supergravity. Actually (see appendix B), the equations of motion of eight dimensional supergravity for our ansatz can be derived from the effective Lagrangian:

$$
\begin{aligned}
\mathcal{L}_{eff} &= e^{3f+\alpha+3h}\Big[\,(f'\,)^2 + (h'\,)^2 - \frac{1}{3}\,(\phi'\,)^2 + 3\,f'\,h' + f'\,\alpha' + \alpha'\,h' \\
&\quad + e^{-2h} + \frac{g^2}{16}\,e^{-2\phi} - \frac{1}{g^2}\,e^{2\phi-4h} - \frac{\Lambda^2}{3}\,e^{-2\alpha\,-\,6h\,-\,2\phi}\Big]\,.
\end{aligned}
\qquad (2.14)
$$

Let us now introduce a new radial variable $\hat{r}$, whose relation to our original coordinate r is given by:

$$
\frac{dr}{d\hat{r}} = e^{-\frac{3}{2}\,h\,-\,\frac{1}{2}\alpha}\,. \qquad (2.15)
$$

The lagrangian in the new variable is $\hat{\mathcal{L}}_{eff} = e^{-\frac{3}{2}\,h\,-\,\frac{1}{2}\alpha}\,\mathcal{L}_{eff}$, where we have taken into account the corresponding jacobian. If we define a new scalar field $\varsigma \equiv f + \frac{1}{2}\,\alpha + \frac{3}{2}\,h$, and let the dot denotes differentiation with respect to $\hat{r}$, the effective lagrangian takes the form:

$$
\hat{\mathcal{L}}_{eff} = e^{c_1\varsigma}\Big[\,c_2\,\dot{\varsigma}^{\,2} - \frac{1}{2}\,G_{ab}\,\dot{\varphi}^a\,\dot{\varphi}^b - V(\varphi)\,\Big]\,, \qquad (2.16)
$$

where $c_1 = 3$, $c_2 = 1$, and φ^a denotes a vector whose components are α, h and ϕ. The non-vanishing elements of the metric G_{ab} are $G_{\alpha\,\alpha} = G_{\alpha\,h} = \frac{1}{2}$, $G_{h\,h} = \frac{5}{2}$ and $G_{\phi\,\phi} = \frac{2}{3}$. The potential $V(\varphi)$ appearing in $\hat{\mathcal{L}}_{eff}$ is:

$$
V(\varphi) = \frac{1}{g^2}\,e^{2\phi\,-\,7h\,-\,\alpha} - \frac{g^2}{16}\,e^{-2\phi\,-\,3h\,-\,\alpha} - e^{-\,5h\,-\,\alpha} + \frac{\Lambda^2}{3}\,e^{-2\phi\,-\,9h\,-\,3\alpha}\,. \qquad (2.17)
$$

For an effective lagrangian as in (2.16) being supersymmetric, the corresponding potential (2.17) must originate from a superpotential W as:

$$V \,=\, \frac{1}{2}\, G^{ab}\, \frac{\partial W}{\partial \varphi^a}\, \frac{\partial W}{\partial \varphi^b} \,-\, \frac{c_1^2}{4c_2}\, W^2 \,, \tag{2.18}$$

where G^{ab} is the inverse metric. In our particular case W must satisfy:

$$V \,=\, \frac{5}{4}\left(\frac{\partial W}{\partial \alpha}\right)^2 + \frac{1}{4}\left(\frac{\partial W}{\partial h}\right)^2 + \frac{3}{4}\left(\frac{\partial W}{\partial \phi}\right)^2 - \frac{1}{2}\,\frac{\partial W}{\partial \alpha}\,\frac{\partial W}{\partial h} \,-\, \frac{9}{4} W^2 \,. \tag{2.19}$$

For the value of V given above (eq.(2.17)) one can check that eq.(2.19) is satisfied by:

$$W \,=\, -\frac{1}{g}\, e^{\phi - \frac{7}{2} h - \frac{\alpha}{2}} \,-\, \frac{g}{4}\, e^{-\phi - \frac{3}{2} h - \frac{\alpha}{2}} \,+\, \frac{\Lambda}{3}\, e^{-\phi - \frac{9}{2} h - \frac{3}{2}\alpha} \,. \tag{2.20}$$

It is now easy to verify that the first-order *domain wall* equations for this superpotential:

$$\begin{aligned} \dot{\varsigma} \,&=\, -\frac{c_1}{2c_2}\, W \,, \\ \dot{\varphi}^a \,&=\, G^{ab}\, \frac{\partial W}{\partial \varphi^b} \,. \end{aligned} \tag{2.21}$$

are exactly the same (when expressed in terms of the old variable r) as those obtained from the supersymmetric variation of the fermionic fields (eqs.(2.13)).

2.3 Integration of the first-order equations

Let us now integrate eqs.(2.13). In order to simplify the expressions that follow, we shall take from now on the coupling constant $g = 1$. It is rather easy to find a particular solution in which $h - \phi$ is constant. First of all, we change variables from r to t, where t is such that:

$$\frac{dr}{dt} \,=\, e^{\phi} \,. \tag{2.22}$$

It follows from the last two equations in (2.13) that, if $h - \phi$ does not depend on t, we must have:

$$\phi(t) \,=\, h(t) \,-\, \frac{1}{2}\, \log(12) \,. \tag{2.23}$$

By using eq.(2.23) it is not difficult to prove that the values f, α and h are:

$$\begin{aligned} f(t) \,&=\, \frac{t}{12} \,-\, \frac{1}{4}\, \log\Big(c + \frac{12\Lambda}{5}\, e^{-\frac{5}{6}t}\Big) \,, \\ \alpha(t) \,&=\, \frac{t}{12} + \frac{1}{4}\, \log\Big(c + \frac{12\Lambda}{5}\, e^{-\frac{5}{6}t}\Big) \,, \\ h(t) \,&=\, \frac{t}{4} + \frac{1}{4}\, \log\Big(c + \frac{12\Lambda}{5}\, e^{-\frac{5}{6}t}\Big) \,, \end{aligned} \tag{2.24}$$

where c is an integration constant. Requiring that $f = \alpha$ for $\Lambda = 0$ we fix this constant to the value $c = 1$. Notice that this is equivalent to a change of variable in the coordinate y.

Let us now consider the general solution of eqs.(2.13) in which $\phi - h$ is not necessarily constant. First, we define the function $x \equiv 12\, e^{2\phi - 2h}$. It follows from the first-order equations (2.13) that the differential equation satisfied by x is:

$$\frac{dx}{dt} = \frac{1}{2}\, x\,(1 - x)\ . \tag{2.25}$$

The solution of this equation is:

$$x = \frac{1}{1 + b\, e^{-\frac{t}{2}}}\ , \tag{2.26}$$

b being an integration constant. Notice that taking $b = 0$ we get the previous particular solution. In general, we get the following relation between ϕ and h:

$$\phi(t) = h(t) - \frac{1}{2}\log(12) - \frac{1}{2}\log\Big(1 + b\, e^{-\frac{t}{2}}\Big)\ . \tag{2.27}$$

In order to integrate completely the system (2.13), let us define a new function $z \equiv e^{3h + \alpha}$. It is not difficult to check that z satisfies the following differential equation:

$$\frac{dz}{dt} = \Big[\frac{1}{2} + \frac{x}{3}\Big]\, z - 2\Lambda\ . \tag{2.28}$$

Since x is a known function of t, we can integrate $z(t)$. To express the result of this integration, let us define the variable s as:

$$s \equiv \Big[\frac{1}{b}\, e^{\frac{t}{2}} + 1\Big]^{\frac{1}{3}}\ , \tag{2.29}$$

and let $J(s)$ be the following indefinite integral:

$$J(s) \equiv -9 \int \frac{ds}{(s^3 - 1)^2}\ . \tag{2.30}$$

By elementary methods one can perform the integration on the right-hand side of (2.30) and obtain an explicit expression for $J(s)$:

$$J(s) = \frac{3s}{s^3 - 1} + 2\sqrt{3}\,\mathrm{arccot}\Big[\frac{1 + 2s}{\sqrt{3}}\Big] - \log\Big(1 + \frac{3s}{(s-1)^2}\Big)\ . \tag{2.31}$$

In terms of $J(s)$, the function z is given by:

$$z = e^{\frac{5t}{6}}\Big(1 + b\, e^{-\frac{t}{2}}\Big)^{\frac{2}{3}}\Big(1 + \tilde{\Lambda}\, J(s)\Big)\ , \tag{2.32}$$

with $\tilde{\Lambda} = \frac{4}{3}\, b^{-\frac{5}{3}}\,\Lambda$. In (2.32) we have fixed the integration constants to reproduce the $b = 0$ solution. From these results it is easy to obtain the remaining functions in (2.13). They are:

$$\begin{aligned}
f(t) &= \frac{t}{12} - \frac{1}{12}\log\Big(1 + b\, e^{-\frac{t}{2}}\Big) - \frac{1}{4}\log\Big(1 + \tilde{\Lambda}\, J(s)\Big)\ ,\\
\alpha(t) &= \frac{t}{12} - \frac{1}{12}\log\Big(1 + b\, e^{-\frac{t}{2}}\Big) + \frac{1}{4}\log\Big(1 + \tilde{\Lambda}\, J(s)\Big)\ ,\\
h(t) &= \frac{t}{4} + \frac{1}{4}\log\Big(1 + b\, e^{-\frac{t}{2}}\Big) + \frac{1}{4}\log\Big(1 + \tilde{\Lambda}\, J(s)\Big)\ ,
\end{aligned} \tag{2.33}$$

while $\phi(t)$ can be obtained from eq. (2.27). In the limit $b \approx 0$, after taking into account that $J \approx \frac{9}{5} b^{\frac{5}{3}} e^{-\frac{5t}{6}}$, one easily verifies that the solution (2.24) is recovered.

2.4 Uplifting to eleven dimensions

Let us now analyze the eleven dimensional background corresponding to the D=8 BPS configurations found above. The uplifting formula for the metric is given in appendix A.

2.4.1 Smeared M2–branes on the tip of a G_2 cone

We shall consider first the particular solution (2.23)–(2.24). It is convenient to change again the radial coordinate, from t to a new coordinate ρ, whose relation is as follows:

$$e^{\frac{t}{2}} = \frac{1}{18} \rho^3 . \tag{2.34}$$

Notice that, clearly, $\rho \geq 0$. By substituting the solution of eqs.(2.23)–(2.24) in eq.(A.5), one gets the following metric in D=11:

$$ds_{11}^2 = [H(\rho)]^{-\frac{2}{3}} dx_{1,2}^2 + [H(\rho)]^{\frac{1}{3}} \Big[dy^2 + d\rho^2 + \rho^2 ds_6^2 \Big] , \tag{2.35}$$

where ds_6^2 is the metric of a compact Einstein manifold, Y_6, with the topology of $S^3 \times S^3$,

$$ds_6^2 = \frac{1}{12} \sum_{i=1}^{3} (w^i)^2 + \frac{1}{9} \sum_{i=1}^{3} (\tilde{w}^i - \frac{1}{2} w^i)^2 , \tag{2.36}$$

whereas $H(\rho)$ is an harmonic function in the transverse seven dimensional cone over Y_6 –whose metric, $d\rho^2 + \rho^2 ds_6^2$, has G_2 holonomy–,

$$H(\rho) = 1 + \frac{k}{\rho^5} . \tag{2.37}$$

with k being:

$$k = \frac{1296}{5} \sqrt{3} \frac{\Lambda}{(12)^{\frac{1}{6}}} . \tag{2.38}$$

As for the 4-form G, we use the uplifting formula [2]:

$$F_{\underline{x^0 x^1 x^2 \rho}} = 2 e^{\frac{4\phi}{3}} G_{\underline{x^0 x^1 x^2 r}} . \tag{2.39}$$

this leading, in curved indices, to the expression

$$F_{x^0 x^1 x^2 \rho} = \epsilon_{x^0 x^1 x^2} \partial_\rho [H(\rho)]^{-1} , \tag{2.40}$$

where $\epsilon_{x^0 x^1 x^2}$ is the completely anti–symmetric Levi–Civita tensor of the 'external' (in the compactification language) Minkowski space. It is clear from the result of the uplifting that

[2]The factor of two is needed to pass from the Salam–Sezgin conventions of eleven dimensional supergravity to the more standard ones. Notice that F is the corresponding 4-form in D=11 supergravity.

our solution corresponds to a smeared distribution of M2–branes in the tip of the singular cone over $S^3 \times S^3$ with a G_2 holonomy metric found in [17, 18]. Notice that the power of ρ in the harmonic function (2.37) is the one expected within this interpretation. Furthermore, notice that the relation between the 4-form and the *warp* factor of the eleven dimensional metric (2.35) is characteristic of M Theory compactifications in eight dimensional manifolds [19].

The somehow unusual appearance of a smeared configuration in this approach deserves some comments. We should first remind that, even in the case of flat D–branes, it is well known that D2–branes have a low energy range, $g_{YM}^2 < U < g_{YM}^2 N^{\frac{1}{5}}$, in which string theory is strongly coupled but the eleven dimensional curvature is small, and the appropriate description is given in terms of the supergravity solution of smeared (in the eleventh circle direction) M2–branes [5]. In other words, the D=11 configuration obtained by uplifting the D2–brane solution is not the standard localized M2–brane. This result also holds in presence of D6–branes. In fact, a system of D2–branes localized on flat D6–branes [3] always has a low energy range described by smeared M2–branes [12]. It is natural to expect that, if the D6–branes are wrapping a supersymmetric cycle, the corresponding description will be given in terms of smeared M2–branes transverse to some special holonomy manifold. When we go further towards the IR, say $U < g_{YM}^2$, we expect the smeared solution to be replaced (resolved) by a periodic array of localized M2–branes along the eleventh circle. Closer enough to the M2–branes, we should recover a conformal field theory. There must be a more physical solution in D=11 supergravity smoothly describing this transition in which, flowing towards the UV, the solution smears before the eleventh circle radius becomes smaller than the eleven dimensional Planck length [5].

2.4.2 Smeared M2–branes on the resolved G_2 cone

The case considered above is singular. The general solution (2.33) obtained before resolves the conical singularity of the transverse G_2 manifold. Let us then uplift that solution (2.33). First of all, we introduce the parameter a, related to the integration constant b of eq.(2.26) as follows, $b = \frac{a^3}{18}$. Moreover, let us further change to a new variable ρ, which is now related to t by means of the expression:

$$e^{\frac{t}{2}} = \frac{1}{18}\,(\,\rho^3 - a^3\,)\;. \tag{2.41}$$

It follows immediately from (2.41) that the range of ρ is $\rho \geq a$. On the other hand, the variable s introduced in eq.(2.29) is, in terms of ρ and a, simply given by $s = \frac{\rho}{a}$. After some elementary calculations, one can verify that the functions f, α, h and ϕ of eqs.(2.27) and (2.33) can be written as:

$$e^{2f} = \frac{\rho}{(18)^{\frac{1}{3}}}\Big(1-\frac{a^3}{\rho^3}\Big)^{\frac{1}{2}}\Big[H(\rho)\Big]^{-\frac{1}{2}}, \qquad e^{2\alpha} = \frac{\rho}{(18)^{\frac{1}{3}}}\Big(1-\frac{a^3}{\rho^3}\Big)^{\frac{1}{2}}\Big[H(\rho)\Big]^{\frac{1}{2}},$$

$$e^{2h} = \frac{\rho^3}{18}\Big(1-\frac{a^3}{\rho^3}\Big)^{\frac{1}{2}}\Big[H(\rho)\Big]^{\frac{1}{2}}, \qquad e^{2\phi} = \frac{\rho^3}{216}\Big(1-\frac{a^3}{\rho^3}\Big)^{\frac{3}{2}}\Big[H(\rho)\Big]^{\frac{1}{2}}, \tag{2.42}$$

[3] Localized intersections and overlappings of D–branes have been studied in [20].

where now the harmonic function $H(\rho)$ is given by:

$$H(\rho)\,=\,e^{2(\alpha-f)}\,=\,1\,+\,\tilde{\Lambda}\,J(\,\frac{\rho}{a}\,)\,\,. \tag{2.43}$$

By using the explicit value of the function J (eq.(2.31)), one can obtain the expression of $H(\rho)$, namely:

$$H(\rho)\,=\,1+\,k\left[\,\frac{5}{3a^3\rho^2}\,\frac{1}{1\,-\,\frac{a^3}{\rho^3}}+\frac{10}{3\sqrt{3}\,a^5}\,\mathrm{arccot}\,[\,\frac{2\rho+a}{a\sqrt{3}}]\,-\,\frac{5}{9a^5}\,\log\,(\,1\,+\,\frac{3a\rho}{(\rho-a)^2})\,\right]\,, \tag{2.44}$$

where the constant k is the same as in eq. (2.38). The uplifted D=11 metric is now of the form:

$$ds^2_{11}\,=\,[\,H(\rho)\,]^{-\frac{2}{3}}\,dx^2_{1,2}\,+\,[\,H(\rho)\,]^{\frac{1}{3}}\,\Big[\,dy^2\,+\,ds^2_7\,\Big]\,\,, \tag{2.45}$$

where ds^2_7 is the metric of a regular manifold of G_2 holonomy found in [17, 18], which is topologically $\mathbb{R}^4\times S^3$, namely:

$$ds^2_7\,=\,\frac{d\rho^2}{1\,-\,\frac{a^3}{\rho^3}}\,+\,\frac{\rho^2}{12}\,\sum_{i=1}^{3}\,(\,w^i)^2\,+\,\frac{\rho^2}{9}\,(\,1\,-\,\frac{a^3}{\rho^3}\,)\,\sum_{i=1}^{3}\,(\,\tilde{w}^i\,-\,\frac{1}{2}\,w^i\,)^2\,\,, \tag{2.46}$$

while the 4-form is still given by (2.40) (with $H(\rho)$ now being the function (2.44)). This solution represents a smeared distribution of M2–branes on the resolved manifold of G_2 holonomy X_7 whose singular limit is the cone over Y_6 obtained above. It is an $\mathbb{R}^4$ bundle over S^3. We see again that the effect of the 4-form flux on the metric is just the introduction of the corresponding warp factors. Actually, the function $H(\rho)$ can also be determined by solving the Laplace equation on the seven dimensional G_2 manifold [21]. It is also interesting to analyze the large and small distance behavior of this harmonic function. When $\rho\rightarrow\infty$, $H(\rho)$ can be approximated as:

$$H(\rho)\approx 1\,+\,\frac{k}{\rho^5}\,+\,\frac{5a^3k}{4\rho^8}\,+\,\cdots\,\,, \tag{2.47}$$

i.e. it has the same leading asymptotic behaviour as the function (2.37). On the other hand, for $\rho\approx a$, $H(\rho)$ diverges as:

$$H(\rho)\approx\frac{5k}{9a^4}\,\frac{1}{\rho-a}\,+\,\frac{10k}{9a^5}\,\log\frac{\rho-a}{a}\,+\,\cdots\,\,. \tag{2.48}$$

It is tempting to argue at this point that this supergravity smeared solution might be the dual of some gauge theory at a given low energy range. The resolution of the conical singularity must render the theory non-conformal in the IR. In order to better understand our solutions, it is important to go to ten dimensions. There are different reductions to type IIA string theory: we can reduce on the smeared direction, or we can embed the M–theory circle in the $\mathbb{R}^4$ fiber or the S^3 base in X_7. We will study them in the following subsection.

2.5 Reduction to D=10 and T-duality

Given an eleven dimensional metric with a Killing vector v, one can generate a background of type IIA D=10 supergravity by means of a Kaluza-Klein dimensional reduction along the direction of v. Actually, if $\partial/\partial z$ is such a Killing vector, the reduction ansatz for the metric is:

$$ds^2_{11} = e^{-\frac{2}{3}\phi_D}\, ds^2_{10} + e^{\frac{4}{3}\phi_D}\,(\,dz + C^{(1)}\,)^2 \,, \tag{2.49}$$

where ϕ_D is the ten-dimensional dilaton and $C^{(1)}$ is the RR potential 1-form of the type IIA theory. In the case of our D=11 metric (2.45), we have several possibilities to choose z.

2.5.1 D2–branes on the tip of a (resolved) G_2 cone

The simplest election –and the most meaningful from the point of view of gauge/string duality, as long as the smearing is removed– is $z = y$, for which the metric and dilaton of the IIA theory are:

$$\begin{aligned} ds^2_{10} &= [\,H(\rho)\,]^{-\frac{1}{2}}\, dx^2_{1,2} + [\,H(\rho)\,]^{\frac{1}{2}}\, ds^2_7 \,, \\ e^{\phi_D} &= [\,H(\rho)\,]^{\frac{1}{4}} \,, \end{aligned} \tag{2.50}$$

while the 4-form field strength of D=11 becomes the RR 4-form $F^{(4)}$ of the type IIA theory and $C^{(1)}$ vanishes. It is clear that this D=10 solution represents a D2 sitting at the tip of the G_2 holonomy manifold X_7, whose principal orbits are topologically trivial $\tilde{S}^3$ bundles over S^3. In the singular limit, when the base S^3 has vanishing volume, we end with D2–branes at the tip of the G_2 cone over the Einstein manifold Y_6. This configuration is reminiscent of the Klebanov–Witten's $D3$–branes placed at the tip of the conifold [22]. Indeed, it is a sort of lower supersymmetric version of it. Notice, however, that the solution resulting from gauged supergravity is the complete D2–brane solution and not its near horizon limit. This might look strange since gauged supergravity usually gives directly the near horizon metric. The reason is that the near horizon limit of the D6–branes (that we would obtain through a different reduction, see below), which are the *host* branes of D=8 gauged supergravity, do not imply, in general, the near horizon limit of the D2–branes that are intersecting them. We will come back to this point later. In summary, in order to get the supergravity dual of the system of D2–branes on the tip of the G_2 cone, we must consider the near horizon limit. We should reintroduce l_p units everywhere and take ρ, a and l_p to zero such that

$$U \equiv \frac{a\rho}{l_p^3} \qquad \text{and} \qquad L \equiv \frac{a^2}{l_p^3} \tag{2.51}$$

are kept fixed. The resulting expression for the harmonic function (2.44), for large U, admits the following asymptotic expansion

$$H(U) = \frac{5\, g^3_{YM}\, N}{3\, l_s^4\, L^3\, U^2} \sum_{n=1}^{\infty} \frac{3n}{3n+2} \Big(\frac{L}{U}\Big)^{3n} \,, \tag{2.52}$$

where $g_{YM}^2 \approx L$ is the three dimensional coupling constant, $al_s^2 = l_p^3$, and N is the number of $D2$–branes. The asymptotic background gives the near horizon limit of N $D2$–branes transverse to the G_2 holonomy manifold:

$$
\begin{aligned}
ds_{10}^2 &= l_s^2 \left(\frac{U^{\frac{5}{2}}}{\sqrt{g_{YM}^2 N}}\, dx_{1,2}^2 + \frac{\sqrt{g_{YM}^2 N}}{U^{\frac{5}{2}}}\, ds_7^2 \right) , \\
e^{\phi_D} &= \left(\frac{g_{YM}^{10} N}{U^5} \right)^{\frac{1}{4}} ,
\end{aligned} \tag{2.53}
$$

and the 4-form field strength F is still given by (2.40). It is analogous to the flat D2–brane [5] except for the fact that the transverse $\mathbb{R}^7$ has been replaced by the G_2 cone over $S^3 \times S^3$. This is the valid description for intermediate high energies, $g_{YM}^2 N > U > g_{YM}^2 N^{\frac{1}{5}}$, where the string coupling and the curvature are small, and the radius of the eleventh circle vanishes.

In the UV we can trust the super Yang–Mills theory description. It is an $\mathcal{N} = 1$ theory in $2+1$ dimensions. We can obtain its field content following the arguments in [22]. In the case of a single $D2$–brane, it is a $U(1) \times U(1)$ gauge theory with four complex scalars Q_i, $\tilde{Q}_i$, $i = 1, 2$, and a vector multiplet whose gauge field can be dualized to a compact scalar that would parametrize the position of the $D2$–branes along the M–theory circle. The vacuum moduli space is given by

$$
|q_1|^2 + |q_2|^2 - |\tilde{q}_1|^2 - |\tilde{q}_2|^2 = L^2 , \tag{2.54}
$$

where q_i, $\tilde{q}_i$ are the scalar components of the superfields Q_i, $\tilde{Q}_i$, which precisely provides an algebraic–geometric description of the manifold X_7 [23].

2.5.2 D2–D6 system wrapping a special Lagrangian S^3

The second possibility we shall explore is the reduction along some compact direction of the G_2 manifold. Let us consider first the three-sphere $\tilde{S}^3$, parametrized by the $su(2)$ left-invariant 1-forms $\tilde{w}^i$. Notice that $\tilde{S}^3$ is external to the D6-brane worldvolume in the D=8 gauged supergravity approach. We shall regard the $\tilde{S}^3$ sphere as a Hopf bundle over a two-sphere, and we will reduce along the fiber of this bundle. Let us denote by $\tilde{\phi}$, $\tilde{\theta}$ and $\tilde{\psi}$ the angles which parametrize the $\tilde{w}^i$'s, as in eq.(2.5) after putting tildes on both sides of the equation. We shall choose $z = \tilde{\psi}$ as the coordinate along which the dimensional reduction will take place. Accordingly [24], let us define the vector $\tilde{\mu}^i$ and the 1-forms $\tilde{e}^i$ by means of the following decomposition of the $\tilde{w}^i$'s:

$$
\tilde{w}^i = \tilde{e}^i + \tilde{\mu}^i \, d\tilde{\psi} . \tag{2.55}
$$

The components of $\tilde{\mu}^i$ and $\tilde{e}^i$ are:

$$
\begin{aligned}
\tilde{\mu}^1 &= \sin\tilde{\theta} \sin\tilde{\phi} , & \tilde{\mu}^2 &= -\sin\tilde{\theta}\cos\tilde{\phi} , & \tilde{\mu}^3 &= \cos\tilde{\theta} , \\
\tilde{e}^1 &= \cos\tilde{\phi}\, d\tilde{\theta} , & \tilde{e}^2 &= \sin\tilde{\phi}\, d\tilde{\theta} , & \tilde{e}^3 &= d\tilde{\phi} .
\end{aligned} \tag{2.56}
$$

Notice that $\tilde{\mu}^i \tilde{\mu}^i = 1$. One can also check the following relation:

$$\tilde{e}^i = \epsilon_{ijk}\, \tilde{\mu}^j\, d\tilde{\mu}^k + \cos\tilde{\theta}\, d\tilde{\phi}\, \tilde{\mu}^i \ , \tag{2.57}$$

from which it follows that $\tilde{e}^i \tilde{\mu}^i = \cos\tilde{\theta}\, d\tilde{\phi}$. Next, let us define the one-forms $D\tilde{\mu}^i$ as:

$$D\tilde{\mu}^i \equiv d\tilde{\mu}^i - \frac{1}{2}\, \epsilon_{ijk}\, w^j\, \tilde{\mu}^k \ . \tag{2.58}$$

It is important to point out that the $D\tilde{\mu}^i$ one-forms are not independent since $\tilde{\mu}^i\, D\tilde{\mu}^i = 0$. Moreover, after some calculation one verifies [24] that:

$$\sum_{i=1}^{3} (\, \tilde{w}^i - \frac{1}{2}\, w^i\,)^2 = \sum_{i=1}^{3} (\, D\tilde{\mu}^i\,)^2 + \sigma^2 \ , \tag{2.59}$$

where σ is given by:

$$\sigma = d\tilde{\psi} + \cos\tilde{\theta}\, d\tilde{\phi} - \frac{1}{2}\, \tilde{\mu}^i\, w^i \ . \tag{2.60}$$

Using eq. (2.59) to rewrite the right-hand side of (2.46), one is able to put the metric (2.45) in the form (2.49) with $z = \tilde{\psi}$. Before giving the form of the resulting D=10 supergravity background, let us write a more explicit expression for $(\, D\tilde{\mu}\,)^2$,

$$\begin{aligned} \sum_{i=1}^{3} (\, D\tilde{\mu}^i\,)^2 \ = \ & \Big(\, d\tilde{\theta} - \cos\tilde{\phi}\, \frac{w^1}{2} - \sin\tilde{\phi}\, \frac{w^2}{2}\, \Big)^2 \\ & + \sin^2\tilde{\theta}\, \Big(\, d\tilde{\phi} + \cot\tilde{\theta}\, \sin\tilde{\phi}\, \frac{w^1}{2} - \cot\tilde{\theta}\, \cos\tilde{\phi}\, \frac{w^2}{2} - \frac{w^3}{2}\, \Big)^2 \ . \end{aligned} \tag{2.61}$$

If we define $\gamma(\rho)$ as:

$$\gamma(\rho) \equiv \frac{\rho^2}{9}\, (\, 1 - \frac{a^3}{\rho^3}\,) \ , \tag{2.62}$$

then, the D=10 metric and dilaton obtained by reducing along $\tilde{\psi}$ are:

$$\begin{aligned} ds^2_{10} &= \Big[\, \frac{\gamma(\rho)}{H(\rho)}\, \Big]^{\frac{1}{2}} \Big[\, dx^2_{1,2} + H(\rho)\, (\, dy^2 + \frac{d\rho^2}{1 - \frac{a^3}{\rho^3}} + \frac{\rho^2}{12} \sum_{i=1}^{3} (\, w^i\,)^2 + \gamma(\rho) \sum_{i=1}^{3} (\, D\tilde{\mu}^i\,)^2\,)\, \Big] \ , \\ e^{\phi_D} &= \Big[\, \gamma(\rho)\, \Big]^{\frac{3}{4}} \Big[\, H(\rho)\, \Big]^{\frac{1}{4}} \ . \end{aligned} \tag{2.63}$$

As the dilaton ϕ_D diverges at $\rho \to \infty$, it follows that this solution has infinite string coupling constant. Moreover, the RR potentials $C^{(1)}$ and $C^{(3)}$ of the type IIA theory are:

$$\begin{aligned} C^{(1)} &= \cos\tilde{\theta}\, d\tilde{\phi} - \frac{1}{2}\, \tilde{\mu}^i\, w^i \ , \\ C^{(3)} &= -\Big[\, H(\rho)\, \Big]^{-1}\, dx^0 \wedge dx^1 \wedge dx^2 \ , \end{aligned} \tag{2.64}$$

whose field strengths are:

$$
\begin{aligned}
F^{(2)} &= -\frac{1}{2}\,\epsilon_{ijk}\,\tilde{\mu}^k \Big[\, D\tilde{\mu}^i \wedge D\tilde{\mu}^j \,+\, \frac{1}{4}\, w^i \wedge w^j \,\Big]\,, \\
F^{(4)} &= \partial_\rho \Big[\, H(\rho)\,\Big]^{-1}\, dx^0 \wedge dx^1 \wedge dx^2 \wedge d\rho\ , \qquad (2.65)
\end{aligned}
$$

which clearly correspond to a (D2-D6)-brane system with the D2–brane smeared in one of the directions of the D6–brane worldvolume (*i.e.* along the y direction). Three of the directions of the D6–brane are wrapping a supersymmetric 3-cycle in a complex deformed Calabi–Yau. Yet, the smearing in D=10 makes this solution a bit awkward from the point of view of the AdS/CFT correspondence. Instead, we can perform a T–duality transformation along that direction.

2.5.3 Curved D3–branes and deformed conifold

Notice that $\partial/\partial y$ is still a Killing vector of the D=10 metric (2.63). Therefore, we can perform a T-duality transformation along the direction of the coordinate y and, in this way, we get the following solution of the type IIB theory:

$$
\begin{aligned}
ds^2_{10} &= \Big[\frac{\gamma(\rho)}{H(\rho)}\Big]^{\frac{1}{2}} \Big[\, dx^2_{1,2} + \frac{dy^2}{\gamma(\rho)} + H(\rho)\,\Big(\frac{d\rho^2}{1-\frac{a^3}{\rho^3}} \,+\, \frac{\rho^2}{12}\sum_{i=1}^{3} (w^i)^2 + \gamma(\rho)\sum_{i=1}^{3}\,(\,D\tilde{\mu}^i\,)^2 \Big)\,\Big]\,, \\
e^{\phi_D} &= \Big[\,\gamma(\rho)\,\Big]^{\frac{1}{2}}\,, \\
F^{(3)} &= \frac{1}{2}\,\epsilon_{ijk}\,\tilde{\mu}^k \Big[\, D\tilde{\mu}^i \wedge D\tilde{\mu}^j \,+\, \frac{1}{4}\, w^i \wedge w^j \,\Big] \wedge dy\,, \\
F^{(5)} &= \partial_\rho \Big[\, H(\rho)\,\Big]^{-1} dx^0 \wedge dx^1 \wedge dx^2 \wedge dy \wedge d\rho \,+\, \text{Hodge dual}\ . \qquad (2.66)
\end{aligned}
$$

The solution (2.66) contains a D3-brane extended along (x^1, x^2, y), with the y-direction distinguished from the other two. For large ρ the space transverse to the D3-brane is topologically a cone over $S^3 \times S^2$. Moreover, since $\gamma(\rho) \to 0$ as $\rho \to a$, the S^2 part of the transverse space shrinks to zero near $\rho = a$ and, thus, the transverse space has the same topology as the deformed conifold.

2.5.4 Type IIA background with RR fluxes

Another possible reduction to the type IIA theory is obtained by choosing the M-theory circle as the Hopf fiber of the three sphere S^3 (the one parametrized by the one-forms w^i). In order to proceed in this way, let us first rewrite the seven dimensional metric (2.46) as:

$$
ds^2_7 = \frac{d\rho^2}{1-\frac{a^3}{\rho^3}} + \frac{\rho^2}{12}\,\xi(\rho)\,\sum_{i=1}^{3}(\,\tilde{w}^i\,)^2 + \beta(\rho)\,\sum_{i=1}^{3}\,(\,w^i - \frac{\xi(\rho)}{2}\,\tilde{w}^i\,)^2\ . \qquad (2.67)
$$

with $\xi(\rho)$ and $\beta(\rho)$ being:

$$\xi(\rho) \equiv \frac{1-\frac{a^3}{\rho^3}}{1-\frac{a^3}{4\rho^3}} \,, \qquad\qquad \beta(\rho) \equiv \frac{\rho^2}{9}\,(\,1 - \frac{a^3}{4\rho^3}\,)\,. \tag{2.68}$$

As in eq.(2.55), we decompose w^i as $w^i = e^i + \mu^i\, d\psi$. The components of e^i and μ^i are similar to the ones written in eq.(2.56). Moreover, if we define the 1-forms $D\mu^i$ as:

$$D\mu^i \equiv d\mu^i - \frac{\xi(\rho)}{2}\,\epsilon_{ijk}\,\tilde{w}^j\,\mu^k \,, \tag{2.69}$$

then, one can easily find expressions of the type of eqs.(2.59)–(2.60) and the D=10 solution is readily obtained. For the metric, dilaton and RR 1-form potential one gets:

$$\begin{aligned}
ds^2_{10} &= \Big[\frac{\beta(\rho)}{H(\rho)}\Big]^{\frac{1}{2}} \Big[\, dx^2_{1,2} + H(\rho)\,(\,dy^2 + \frac{d\rho^2}{1-\frac{a^3}{\rho^3}} + \frac{\rho^2}{12}\,\xi(\rho)\,(\,\tilde{w}^i\,)^2 + \beta(\rho)\,(\,D\mu^i\,)^2\,)\,\Big]\,,\\
e^{\phi_D} &= \Big[\beta(\rho)\Big]^{\frac{3}{4}}\Big[H(\rho)\Big]^{\frac{1}{4}}\,,\\
C^{(1)} &= \cos\theta\, d\phi - \frac{\xi(\rho)}{2}\,\mu^i\,\tilde{w}^i\,,
\end{aligned} \tag{2.70}$$

while the RR potential $C^{(3)}$ is the same as in eq. (2.64).

2.5.5 Curved D3–branes and resolved conifold

We can make a T-duality transformation to the background (2.70) in the direction of the coordinate y. The resulting metric and dilaton are:

$$\begin{aligned}
ds^2_{10} &= \Big[\frac{\beta(\rho)}{H(\rho)}\Big]^{\frac{1}{2}} \Big[\, dx^2_{1,2} + \frac{dy^2}{\beta(\rho)} + H(\rho)\,(\,\frac{d\rho^2}{1-\frac{a^3}{\rho^3}} + \frac{\rho^2}{12}\,\xi(\rho)\,(\,\tilde{w}^i\,)^2 + \beta(\rho)\,(\,D\mu^i\,)^2\,)\,\Big]\,,\\
e^{\phi_D} &= \Big[\beta(\rho)\Big]^{\frac{1}{2}}\,,
\end{aligned} \tag{2.71}$$

which for large ρ corresponds, again, to a D3-brane with a transverse space with the topology of a cone over $S^3\times S^2$. However, since $\xi(\rho)$ vanishes at $\rho = a$, in this case the S^3 part of the cone shrinks to zero as $\rho\to a$ and, therefore, the transverse space has a structure similar to the resolved conifold.

3 D6-branes wrapped on a 2-sphere with 4-form

3.1 First-order equations from SUSY

In this section we will analyze the situation in which the D6–branes are wrapped on a holomorphic two sphere and a four-form flux is turned on along some of the unwrapped directions of its worldvolume. As argued in ref.[7], one must excite in this case a real scalar field –which parametrizes the Coulomb branch of the theory–, out of the coset scalars L^i_α (see appendix A). We then adopt the following ansatz for these scalars [7]:

$$L^i_\alpha \,=\, \mathrm{diag}\,(e^{\lambda}\,,\, e^{\lambda}\,,\, e^{-2\lambda})\;. \tag{3.1}$$

On the other hand, when the D6-brane wraps an S^2, two of the unwrapped directions of its worldvolume are distinguished from the others by the flux of the four-form. Thus, the natural ansatz for the metric in this case is:

$$ds^2_8 \,=\, e^{2f}\,dx^2_{1,2} \,+\, e^{2\alpha}\,dy^2_2 \,+\, e^{2h}\,d\Omega^2_2 \,+\, dr^2\;, \tag{3.2}$$

where $d\Omega^2_2$ is the metric of the unit S^2 and $dy^2_2 = (dy^1)^2 + (dy^2)^2$. For the metric (3.2), the equation of motion of the four-form is satisfied if one adopts the following ansatz for G:

$$G_{\underline{x^0x^1x^2r}} \,=\, \Lambda\, e^{-2\alpha-2h-2\phi}\;, \tag{3.3}$$

where, as before, Λ is a constant. The BPS configurations for our ansatz can be obtained by requiring the vanishing of the supersymmetry variations of the fermionic fields. To find these configurations we must determine first the spinor projections and the gauge field which implement the appropriate topological twisting. In order to specify them, let us represent the S^2 line element in terms of two angles θ^1 and ϕ^1 as $d\Omega^2_2 \,=\, (\,d\theta^1\,)^2 + (\,\sin\theta^1\,)^2\,(\,d\phi^1\,)^2$. The gauge field potential A^i which we will consider has only components along the direction $i = 3$, its field strength being given by the volume form of S^2 [7],

$$A^3 \,=\, \frac{1}{g}\,\cos\theta^1 d\phi^1\;, \tag{3.4}$$

while the corresponding spinor projections are the ones in [7] plus an extra projection related to the presence of a G flux:

$$\begin{aligned}
&(\,\gamma_{\underline{\theta^1\phi^1}}\otimes \mathbb{I}\,)\epsilon \,=\, -(\,\mathbb{I}\otimes\sigma^1\,\sigma^2)\epsilon\;, \qquad\qquad (\,\gamma_{\underline{r}}\otimes \mathbb{I}\,)\,\epsilon = \,-i(\,\gamma_9\otimes\mathbb{I}\,)\epsilon\,,\\
&(\,\gamma_{\underline{x^0x^1x^2}}\,\otimes \mathbb{I}\,)\epsilon \,=\, -\epsilon\;.
\end{aligned} \tag{3.5}$$

The number of unbroken supercharges is then four. It is now straightforward to find the first-order equations which follow from the conditions $\delta\psi_\lambda = \delta\chi_i = 0$. One gets:

$$\begin{aligned}
f' &= -\frac{1}{6g}\,e^{\phi-2h-2\lambda} \,+\, \frac{g}{24}\,e^{-\phi}\,(\,2e^{2\lambda} \,+\, e^{-4\lambda}\,) \,+\, \frac{\Lambda}{2}\,e^{-\phi-2h-2\alpha}\;,\\
\alpha' &= -\frac{1}{6g}\,e^{\phi-2h-2\lambda} \,+\, \frac{g}{24}\,e^{-\phi}\,(\,2e^{2\lambda} \,+\, e^{-4\lambda}\,) \,-\, \frac{\Lambda}{2}\,e^{-\phi-2h-2\alpha}\;,\\
h' &= \frac{5}{6g}\,e^{\phi-2h-2\lambda} \,+\, \frac{g}{24}\,e^{-\phi}\,(\,2e^{2\lambda} \,+\, e^{-4\lambda}\,) \,-\, \frac{\Lambda}{2}\,e^{-\phi-2h-2\alpha}\;,\\
\phi' &= -\frac{1}{2g}\,e^{\phi-2h-2\lambda} \,+\, \frac{g}{8}\,e^{-\phi}\,(\,2e^{2\lambda} \,+\, e^{-4\lambda}\,) \,-\, \frac{\Lambda}{2}\,e^{-\phi-2h-2\alpha}\;,\\
\lambda' &= \frac{1}{3g}\,e^{\phi-2h-2\lambda} \,-\, \frac{g}{6}\,e^{-\phi}\,(\,e^{2\lambda} \,-\, e^{-4\lambda}\,)\;.
\end{aligned} \tag{3.6}$$

As a check, it is interesting to verify in eq. (3.6) that, when $\Lambda = 0$, $f' = \alpha' = \phi'/3$, and the resulting equations coincide with those of ref.[7].

3.2 First-order equations from a superpotential

It is also possible in this case to give another derivation of the first-order system (3.6) by analyzing the effective lagrangian for our ansatz. Let us briefly present it here for completeness. After some calculations one can verify that this effective lagrangian is:

$$\begin{aligned} \mathcal{L}_{eff} \;=\; & e^{3f+2\alpha+2h}\Big[\,\frac{3}{2}\,(f\,')^2\,+\,\frac{1}{2}\,(\alpha\,')^2\,+\,\frac{1}{2}\,(h\,')^2\,-\,\frac{3}{2}\,(\lambda\,')^2\,-\,\frac{1}{2}\,(\phi\,')^2 \\ & +3f\,'\alpha\,'\,+\,3f\,'h\,'\,+\,2h'\alpha\,'\,+\,\frac{1}{2}\,e^{-2h}\,+\,\frac{g^2}{16}\,e^{-2\phi}\,(\,2e^{-2\lambda}\,-\,\frac{1}{2}\,e^{-8\lambda}\,) \\ & -\,\frac{1}{2g^2}\,e^{2\phi-4h-4\lambda}\,-\,\frac{\Lambda^2}{2}\,e^{-4\alpha-2\phi-4h}\,\Big]\,. \end{aligned} \tag{3.7}$$

As in section 2.2, let us now define a new variable $\hat{r}$ as:

$$\frac{dr}{d\hat{r}}\,=\,e^{-h\,-\,\alpha}\;. \tag{3.8}$$

After taking into account the jacobian for the change of variable (3.8), one concludes that the effective lagrangian in the new variable is $\hat{\mathcal{L}}_{eff}\,=\,e^{-h\,-\,\alpha}\,\mathcal{L}_{eff}$. Moreover, if the dot denotes differentiation with respect to $\hat{r}$, it is easy to check that $\hat{\mathcal{L}}_{eff}$ can be put in the form (2.16), with $\varsigma\,\equiv\,f\,+\,h\,+\,\alpha$. In eq.(2.16), the constants c_1 and c_2 become $c_1=3$, $c_2=3/2$, and now φ^a has four components, namely, $\varphi^a\,=\,(\alpha,h,\phi,\lambda)$. The non-vanishing elements of the metric G_{ab} are $G_{\alpha\,\alpha}\,=\,G_{h\,h}\,=\,2$, $G_{\alpha\,h}\,=\,G_{\phi\,\phi}\,=\,1$ and $G_{\lambda\,\lambda}\,=\,3$, and the potential V is given by:

$$V=\frac{1}{2g^2}\,e^{2\phi-6h-2\alpha-4\lambda}+\frac{g^2}{32}\,e^{-2\phi-2h-2\alpha}\,(\,e^{-8\lambda}\,-\,4e^{-2\lambda}\,)-\frac{1}{2}\,e^{-4h-2\alpha}+\frac{\Lambda^2}{2}\,e^{-6\alpha-2\phi-6h}\;. \tag{3.9}$$

The corresponding superpotential W must satisfy eq. (2.18), which in this case becomes:

$$V=\frac{1}{3}\left(\frac{\partial W}{\partial\alpha}\right)^2+\frac{1}{3}\left(\frac{\partial W}{\partial h}\right)^2+\frac{1}{2}\left(\frac{\partial W}{\partial\phi}\right)^2+\frac{1}{6}\left(\frac{\partial W}{\partial\lambda}\right)^2-\frac{1}{3}\,\frac{\partial W}{\partial h}\,\frac{\partial W}{\partial\alpha}\,-\,\frac{3}{2}\,W^2\;. \tag{3.10}$$

After some elementary calculation, one can prove that W can be taken as:

$$W=-\frac{1}{2g}\,e^{\phi\,-\,3h\,-\,\alpha\,-\,2\lambda}\,-\,\frac{g}{8}\,e^{-\phi\,-\,h\,-\,\alpha}\,(\,e^{-4\lambda}\,+\,2e^{2\lambda}\,)\,+\,\frac{\Lambda}{2}\,e^{-3\alpha-3h-\phi}\;. \tag{3.11}$$

The first-order equations for this superpotential can be obtained by substituting (3.11) on the right-hand side of eq.(2.21). It is not difficult to check that, in terms of the original variable r, one gets exactly the first-order system (3.6).

3.3 Integration of the first-order equations

We now undertake the task of integrating the system (3.6). As in section 2.3, we will take $g\,=\,1$ from now on and we shall begin by a simpler particular case, in which some combinations of the unknown functions are constant. By inspecting (3.6) one easily realizes that λ can be kept constant if $\phi - h$ is also constant. Actually, in this case one must have:

$$\lambda = \frac{1}{6}\log\Big(\frac{3}{2}\Big), \qquad \phi = h - \frac{1}{6}\log(96), \tag{3.12}$$

as in the singular solution found in [7] when $\Lambda = 0$. In order to integrate completely the system in this particular case, let us change variables from r to t, where t is determined by the condition:

$$\frac{dr}{dt} = e^{\phi + 4\lambda}. \tag{3.13}$$

Then, defining $\hat{\Lambda} = 2\big(\frac{3}{2}\big)^{\frac{2}{3}}\Lambda$, we find the following solution:

$$\begin{aligned} f(t) &= \frac{1}{8}t - \frac{1}{4}\log\Big(1 + \hat{\Lambda}\,e^{-t}\Big), \\ \alpha(t) &= \frac{1}{8}t + \frac{1}{4}\log\Big(1 + \hat{\Lambda}\,e^{-t}\Big), \\ h(t) &= \frac{3}{8}t + \frac{1}{4}\log\Big(1 + \hat{\Lambda}\,e^{-t}\Big), \end{aligned} \tag{3.14}$$

where we have fixed the integration constants by imposing $f = \alpha$ for $\Lambda = 0$. Notice that ϕ, in this solution, can be obtained from (3.12) and (3.14).

Let us now find a general solution of (3.6). First of all, we define the function $x \equiv 4e^{2\phi - 2h + 2\lambda}$. It can be easily verified that x satisfies the differential equation (2.25), where now t is the variable defined in (3.13). We write the integral of eq. (2.25) as:

$$x = \frac{1}{1 + c e^{-\frac{t}{2}}}, \tag{3.15}$$

with c being an integration constant. It follows from the first-order system (3.6) that λ satisfies the equation:

$$\frac{d\lambda}{dt} = \frac{1}{6}(1 - e^{6\lambda}) + \frac{x}{12}. \tag{3.16}$$

By using the explicit dependence of x on t, displayed in eq. (3.15), the integral of eq. (3.16) is easy to find. In order to express this integral in a convenient way, let us parametrize λ as:

$$\lambda = \frac{1}{6}\Big[\log\big(\frac{3}{2}\big) - \log\kappa\Big]. \tag{3.17}$$

Notice that $\kappa = 1$ corresponds to the solution (3.12), in which λ is not running and $x = 1$. In general, the function $\kappa(t)$ is given by:

$$\kappa(t) = \frac{e^{\frac{3}{2}t} + \frac{3}{2}c\,e^{t} + d}{e^{\frac{3}{2}t} + c\,e^{t}}, \tag{3.18}$$

where d is a new integration constant. Next, let us define the function z as $z \equiv e^{2(\alpha + h)}$. After simple manipulations of the system (3.6), one reaches the conclusion that z satisfies the equation:

$$\frac{dz}{dt} = \Big[\frac{x}{3} + \frac{1}{6}(2e^{6\lambda} + 1)\Big]z - 2\Lambda\,e^{4\lambda}, \tag{3.19}$$

which can be solved by using the explicit dependence of x and λ on t. To express the solutions of this equation, let us define the integral:

$$I(t) \equiv - \int \frac{dt}{e^t \left(1 + \frac{3}{2} c\, e^{-\frac{t}{2}} + d\, e^{-\frac{3t}{2}} \right)} \ . \tag{3.20}$$

Then, one has:

$$z = e^t \left(1 + c\, e^{-\frac{t}{2}} \right) [\, \kappa(t) \,]^{\frac{1}{3}} \left(1 + \hat{\Lambda} I(t) \right) \ , \tag{3.21}$$

with $\hat{\Lambda}$ the same as in eq. (3.14). Notice that in (3.21) we have fixed the integration constant to have the same value of z as in the solution (3.14) when $c = d = 0$. Once x, λ and z are known, the functions f, α, h and ϕ can be obtained by direct integration of the equations:

$$\begin{aligned}
\frac{df}{dt} &= -\frac{x}{24} + \frac{1}{24} \left(2e^{6\lambda} + 1 \right) + \frac{\Lambda}{2} \frac{e^{4\lambda}}{z} \ , \\
\frac{d\alpha}{dt} &= -\frac{x}{24} + \frac{1}{24} \left(2e^{6\lambda} + 1 \right) - \frac{\Lambda}{2} \frac{e^{4\lambda}}{z} \ , \\
\frac{dh}{dt} &= \frac{5x}{24} + \frac{1}{24} \left(2e^{6\lambda} + 1 \right) - \frac{\Lambda}{2} \frac{e^{4\lambda}}{z} \ , \\
\frac{d\phi}{dt} &= -\frac{x}{8} + \frac{1}{8} \left(2e^{6\lambda} + 1 \right) - \frac{\Lambda}{2} \frac{e^{4\lambda}}{z} \ .
\end{aligned} \tag{3.22}$$

The result is given as follows:

$$\begin{aligned}
f(t) &= \frac{t}{8} + \frac{1}{12} \log \kappa(t) - \frac{1}{4} \log \left(1 + \hat{\Lambda} I(t) \right) \ , \\
\alpha(t) &= \frac{t}{8} + \frac{1}{12} \log \kappa(t) + \frac{1}{4} \log \left(1 + \hat{\Lambda} I(t) \right) \ , \\
h(t) &= \frac{3t}{8} + \frac{1}{2} \log \left(1 + c e^{-\frac{t}{2}} \right) + \frac{1}{12} \log \kappa(t) + \frac{1}{4} \log \left(1 + \hat{\Lambda} I(t) \right) \ , \\
\phi(t) &= \frac{3t}{8} + \frac{1}{4} \log \kappa(t) + \frac{1}{4} \log \left(1 + \hat{\Lambda} I(t) \right) - \frac{1}{6} \log(96) \ ,
\end{aligned} \tag{3.23}$$

where, again, we have fixed the integration constants in order to reproduce the solution (3.14) when $c = d = 0$.

3.4 Uplifting to eleven dimensions

3.4.1 Smeared M2–branes at the tip of the conifold

Let us consider first the uplifting to eleven dimensions of the particular solution (3.14). Introducing a new radial coordinate ρ as:

$$e^{\frac{t}{2}} = \frac{1}{6(96)^{\frac{1}{9}}} \, \rho^2 \ , \tag{3.24}$$

we get that the corresponding eleven dimensional metric takes the form:

$$ds_{11}^2 = [H(\rho)]^{-\frac{2}{3}} dx_{1,2}^2 + [H(\rho)]^{\frac{1}{3}} \Big[dy_2^2 + d\rho^2 + \rho^2 ds_5^2 \Big] , \tag{3.25}$$

where now the harmonic function $H(\rho)$ is:

$$H(\rho) = 1 + \frac{k}{\rho^4} , \tag{3.26}$$

and the constant k is related to Λ by means of the expression:

$$k = 432 \frac{\Lambda}{(96)^{\frac{1}{9}}} . \tag{3.27}$$

The metric ds_5^2 appearing in eq.(3.25) is the one corresponding to the Einstein $T^{1,1}$ space, namely:

$$ds_5^2 = \frac{1}{9} (d\psi + \sum_{a=1,2} \cos\theta_a \, d\phi_a)^2 + \frac{1}{6} \sum_{a=1,2} (d\theta_a^2 + \sin^2\theta_a \, d\phi_a^2) . \tag{3.28}$$

Recall that $d\rho^2 + \rho^2 ds_5^2$ is the metric of the singular conifold with base $T^{1,1}$ [25]. The 4-form F can be obtained by plugging the solution (3.14) into the uplifting formula (2.39). The result is just (2.40), where now the harmonic function $H(\rho)$ is the one given in (3.26). It follows from these results that this solution can be interpreted as the geometry created by a smeared distribution of M2-branes located at the tip of the singular conifold. Notice that we are now smearing the M2- brane along two coordinates, which agrees with the power of ρ in eq. (3.26)

3.4.2 Smeared M2–branes and generalized resolved conifold

Let us now uplift the general solution (3.23). First of all we change variables from t to ρ as in eq.(3.24). Then, let us represent the constants c and d in terms of two parameters a and b as follows:

$$c = \frac{1}{(96)^{\frac{1}{9}}} a^2 , \qquad d = -\frac{1}{6^3 (96)^{\frac{1}{3}}} b^6 . \tag{3.29}$$

(we are assuming that $d \leq 0$). With these definitions the function κ becomes:

$$\kappa(\rho) = \frac{\rho^6 + 9a^2 \rho^4 - b^6}{\rho^6 + 6a^2 \rho^4} , \tag{3.30}$$

and, if we define now:

$$H(\rho) = 1 + \hat{\Lambda} \, I(t(\rho)) , \tag{3.31}$$

where $I(t(\rho))$ is the integral defined in eq.(3.20), then the D=11 metric can be written as:

$$ds_{11}^2 = [H(\rho)]^{-\frac{2}{3}} dx_{1,2}^2 + [H(\rho)]^{\frac{1}{3}} \Big[dy_2^2 + ds_6^2 \Big] , \tag{3.32}$$

where the six dimensional metric ds_6^2 is given by:

$$
\begin{aligned}
ds_6^2 \;&=\; [\,\kappa(\rho)\,]^{-1}\,d\rho^2 \,+\, \frac{\rho^2}{9}\,\kappa(\rho)\,(\,d\psi \,+\, \sum_{a=1,2}\cos\theta_a\,d\phi_a\,)^2 \,+ \\
&\qquad +\frac{1}{6}\,(\,\rho^2 \,+\, 6a^2\,)\,(\,d\theta_1^2 \,+\, \sin^2\theta_1\,d\phi_1^2\,) \,+\, \frac{1}{6}\,\rho^2\,(\,d\theta_2^2 \,+\, \sin^2\theta_2\,d\phi_2^2\,)\;. \qquad (3.33)
\end{aligned}
$$

Moreover, by performing the change of variables (3.24) in (3.20), we obtain the following integral representation of the harmonic function $H(\rho)$:

$$
H(\rho) \,=\, 1 \,+\, 4k\int_{\rho}^{\infty} \frac{\tau d\tau}{\tau^6 \,+\, 9a^2\tau^4 \,-\, b^6}\;, \qquad (3.34)
$$

with k given by eq. (3.27). On the other hand, the 4-from F for this solution can be put in the form (2.40) with $H(\rho)$ given by eq.(3.34).

The six dimensional metric (3.33) is the one corresponding to the small resolution of the generalized conifold [25, 27, 26, 28]. The parameter b was introduced in refs.[27, 28], where it was pointed out that for $a=0$ and $b>0$ the metric (3.33) can be made non-singular if one takes $\rho \geq b$ and makes a $\mathbb{Z}_2$ identification of the fiber coordinate ψ. In the context of D=8 gauged supergravity, the $b=0$ metric was obtained in ref.[7]. Our result generalizes the one in [7], even in the absence of 4-form flux, and shows that eight dimensional gauged supergravity can easily incorporate the two-parameter metric of refs.[27, 28]. Moreover, as we have switched on the 4-form F in our solution, the corresponding metric contains the appropriate powers of the harmonic function $H(\rho)$, whose integral representation is given in eq.(3.34). It is immediate to conclude from this representation that $H(\rho)$ behaves for $\rho\rightarrow\infty$ exactly as the right-hand side of eq.(3.26). In order to find out the behaviour of H at small ρ, let us perform explicitly the integral (3.34) in some particular cases. First of all, we consider the case $b=0$, for which $H(\rho)$ is given by:

$$
H(\rho) \,=\, 1 \,+\, \frac{2k}{9a^2}\,\frac{1}{\rho^2} \,-\, \frac{2k}{81a^4}\,\log\,(\,1 \,+\, \frac{9a^2}{\rho^2}\,)\;, \qquad (b=0)\;. \qquad (3.35)
$$

This expression for $H(\rho)$ coincides exactly with the one found in [26] for the case of a D3-brane at the tip of the small resolution of the conifold, which can be obtained from our solution by dimensional reduction and T-duality (see below). For $\rho\approx 0$ the harmonic function behaves as:

$$
H(\rho) \,\approx\, \frac{2k}{9a^2}\,\frac{1}{\rho^2}\;, \qquad (b=0)\;. \qquad (3.36)
$$

When $a=0$ the integral (3.34) can also explicitly performed , with the result:

$$
H(\rho) \,=\, 1 \,-\, \frac{2k}{b^4}\,\Big[\,\frac{1}{6}\,\log\frac{(\rho^2-b^2)^3}{\rho^6-b^6} \,+\, \frac{1}{\sqrt{3}}\,\mathrm{arccot}\frac{2\rho^2+b^2}{\sqrt{3}\,b^2}\,\Big]\;, \qquad (a=0)\;, \qquad (3.37)
$$

and, again, this result coincides with that of ref.[28]. For $\rho\approx b$ the function in (3.37) has a logarithmic behaviour of the form:

$$
H(\rho) \,\approx\, -\frac{2k}{3b^4}\,\log\frac{\rho-b}{b}\;, \qquad (a=0)\;. \qquad (3.38)
$$

For general values of a and b the integral (3.34) can be performed by factorizing the polynomial in the denominator. The result depends on the sign of the "discriminant" $\Delta = b^6 - 108\, a^6$. The analysis of the different cases has been carried out in ref. [28] and will not be repeated here.

3.5 Reduction to D=10 and T-duality

As in section 2.5 we can dimensionally reduce and T-dualize the metric (3.32) along different directions.

3.5.1 D3–branes at the tip of the generalized resolved conifold

Let us consider first a reduction along a direction orthogonal to the six dimensional metric (3.33). Notice that $\partial/\partial y^1$ and $\partial/\partial y^2$ are Killing vectors of (3.32). Let us reduce along y^2 followed by a T-duality transformation along y^1. The resulting metric in the IIB theory is:

$$ds^2_{10} = [\, H(\rho)\,]^{-\frac{1}{2}} \Big[dx^2_{1,2} + (dy^1)^2 \Big] + [\, H(\rho)\,]^{\frac{1}{2}}\, ds^2_6 \,, \tag{3.39}$$

while the dilaton is constant and there is a RR 5-form:

$$F^{(5)} = \partial_\rho \Big[\, H(\rho) \Big]^{-1} dx^0 \wedge dx^1 \wedge dx^2 \wedge dy \wedge d\rho \; + \; \text{Hodge dual} \,. \tag{3.40}$$

This solution is precisely the one studied in ref.[28] and corresponds to a D3–brane located at the tip of the generalized resolved conifold.

3.5.2 Smeared D2–D6 wrapped on a 2-cycle

Another possibility is to reduce along the fiber ψ of the $T^{1,1}$ space. In order to write the result of this reduction, let us define the function:

$$\Gamma(\rho) \equiv \frac{\rho^2}{9}\, \kappa(\rho) \,. \tag{3.41}$$

Then, the solution of the type IIA theory that one obtains by reducing along ψ is:

$$\begin{aligned}
ds^2_{10} &= \left[\frac{\Gamma(\rho)}{H(\rho)} \right]^{\frac{1}{2}} \Big[dx^2_{1,2} + H(\rho) \,(\, dy_2^2 + \frac{d\rho^2}{\kappa(\rho)} + \frac{\rho^2 + 6a^2}{6}\, d\Omega^2_{2,1} + \frac{\rho^2}{6}\, d\Omega^2_{2,2} \,) \Big] \,, \\
e^{\phi_D} &= \Big[\, \Gamma(\rho) \Big]^{\frac{3}{4}} \Big[\, H(\rho) \Big]^{\frac{1}{4}} \,, \\
F^{(2)} &= \epsilon^1_{(2)} + \epsilon^2_{(2)} \,, \\
F^{(4)} &= \partial_\rho \Big[\, H(\rho) \Big]^{-1} dx^0 \wedge dx^1 \wedge dx^2 \wedge d\rho \,,
\end{aligned} \tag{3.42}$$

where $d\Omega^2_{2,a} = d\theta_a^2 + \sin^2\theta_a d\phi_a^2$ and $\epsilon^a_{(2)} = \sin\theta_a d\phi_a \wedge d\theta_a$ for $a = 1, 2$. This solution corresponds to a system of (D2-D6)-branes, with the D2-brane extended along (x^1, x^2) and smeared in (y^1, y^2) and the D6-brane wrapped on a two cycle.

3.5.3 D4–branes

If we now perform T-duality transformations along the coordinates (y^1, y^2), we arrive at a system composed by D4-branes, for which the metric and dilaton are:

$$
\begin{aligned}
ds^2_{10} &= \left[\frac{\Gamma(\rho)}{H(\rho)}\right]^{\frac{1}{2}} \Big[\, dx^2_{1,2} + \frac{dy_2^2}{\Gamma(\rho)} + H(\rho)\,(\,\frac{d\rho^2}{\kappa(\rho)} + \frac{\rho^2+6a^2}{6}\, d\Omega^2_{2,1} + \frac{\rho^2}{6}\, d\Omega^2_{2,2}\,)\,\Big]\;,\\
e^{\phi_D} &= \left[\frac{\Gamma(\rho)}{H(\rho)}\right]^{\frac{1}{4}}\;.
\end{aligned}
\tag{3.43}
$$

Moreover, the direct application of the T-duality rules gives the following RR potentials:

$$
\begin{aligned}
C^{(3)} &= \cos\theta_1\, d\phi_1 \wedge dy^1 \wedge dy^2 \,+\, \cos\theta_2\, d\phi_2 \wedge dy^1 \wedge dy^2\;,\\
C^{(5)} &= [\,H(\rho)\,]^{-1}\, dx^0 \wedge dx^1 \wedge dx^2 \wedge dy^1 \wedge dy^2\;.
\end{aligned}
\tag{3.44}
$$

However, since $C^{(5)}$ is really the potential of $F^{(6)} = {}^*F^{(4)}$, we will only have a four-form RR field strength, given by:

$$
F^{(4)} = (\,\epsilon^1_{(2)} + \epsilon^2_{(2)}\,) \wedge dy^1 \wedge dy^2 + \frac{k}{27}\,\epsilon^1_{(2)} \wedge \epsilon^2_{(2)}\;,
\tag{3.45}
$$

where k is the constant appearing in the harmonic function $H(\rho)$.

4 Summary and Conclusions

In this paper we have studied supergravity solutions corresponding to D6-branes which wrap two- and three-cycles and have a four-form flux along the non compact directions of their worldvolume. These solutions are found first in eight-dimensional gauged supergravity by solving a system of first-order equations which arises by requiring that the solution be supersymmetric or, equivalently, by deriving them from a superpotential in the corresponding effective lagrangian problem. After uplifting them to eleven dimensions, our solutions give rise to geometries which are the small resolution of the conifold (for D6-branes wrapping a two cycle) or a manifold of G_2 holonomy (in the case of a D6-brane wrapping an S^3 in T^*S^3), with the corresponding warp factors included. The latter are the effect of the four-form flux on the metric, a fact which we have checked in general in appendix C. These configurations can be interpreted as smeared M2-branes on the tip of a (resolved) cone. By performing different Kaluza-Klein reductions and T-dualities we have obtained several solutions corresponding to D2, D2-D6, D3 and D4 systems and, in some cases, we have discussed the corresponding field theory duals in 2+1 dimensions.

Let us finally point out some directions which would be worth to explore in future. First of all, it is clear that it would be desirable to have a better understanding of the field theory duals to the supergravity solutions studied here. Moreover, it would be also interesting to look at solutions which also have non-vanishing Neveu-Schwarz fluxes, as those studied in ref. [29]. Another interesting problem to look at is the generation, in the framework of D=8 gauged supergravity, of solutions with non-vanishing components of the 4-form along the compact directions of the special holonomy manifold. The corresponding field theory duals would be three-dimensional gauge theories with Chern-Simons terms. We could also try to generalize our ansatzs (2.2) and (3.2) for the metric to the case in which the D2-brane is localized in the y direction. We hope to report on these issues in a near future.

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Appendices

A D=8 gauged supergravity

The maximal eight dimensional gauged supergravity was constructed by Salam and Sezgin in ref. [15] by means of a dimensional reduction of D=11 supergravity on a $SU(2)$ group manifold [11]. The bosonic field content of this theory can be consistently truncated to include the metric $g_{\mu\nu}$, a dilatonic scalar ϕ, five scalars parametrized by a 3×3 unimodular matrix L^i_α which lives in the coset $SL(3,\mathbb{R})/SO(3)$, an $SU(2)$ gauge potential A^i_μ and a three-form potential B. The kinetic energy of the coset scalars L^i_α is given in terms of the symmetric traceless matrix $P_{\mu\, ij}$ defined by means of the expression:

$$P_{\mu\, ij} + Q_{\mu\, ij} \;=\; L^\alpha_i \,(\,\partial_\mu\, \delta^\beta_\alpha \,-\, g\,\epsilon_{\alpha\beta\gamma}\, A^\gamma_\mu\,)\, L_{\beta j}\;, \tag{A.1}$$

where $Q_{\mu\, ij}$ is, by definition, the antisymmetric part of the right-hand side of eq. (A.1). Furthermore, the potential energy of the coset scalars is written in terms of the so-called T-tensor, T^{ij}, and of its trace, T, defined as:

$$T^{ij} \;=\; L^i_\alpha\, L^i_\beta\, \delta^{\alpha\beta}\;, \qquad\qquad T \;=\; \delta_{ij}\, T^{ij}\;. \tag{A.2}$$

If $F^i_{\mu\nu}$ is the field strength of the $SU(2)$ gauge field A^i_μ and if $G_{\mu\nu\rho\sigma}$ denotes the components of dB, the bosonic lagrangian for this truncation of D=8 gauged supergravity is:

$$\mathcal{L} \;=\; \sqrt{-g_{(8)}}\;\Big[\,\frac{1}{4}\,R \,-\, \frac{1}{4}\,e^{2\phi}\,F^i_{\mu\nu}\,F^{\mu\nu\; i} \,-\, \frac{1}{4}\,P_{\mu\, ij}\,P^{\mu\, ij} \,-\, \frac{1}{2}\,\partial_\mu\,\phi\partial^\mu\,\phi \,-$$

$$-\frac{g^2}{16}\, e^{-2\phi}\,(\, T_{ij}\, T^{ij} \,-\, \frac{1}{2}T^2\,) \,-\, \frac{1}{48}\, e^{2\phi}\, G_{\mu\nu\rho\sigma}\, G^{\mu\nu\rho\sigma} \,\Big]\;, \tag{A.3}$$

and the corresponding equations of motion are:

$$\begin{aligned}
R_{\mu\nu} &= P_{\mu\, ij}P_{\nu}^{ij} \,+\, 2\partial_\mu\phi\partial_\nu\phi \,+\, 2e^{2\phi}\, F^i_{\mu\lambda}\, F_\nu^{\lambda\, i} \,-\, \frac{1}{3}\, g_{\mu\nu}\nabla^2\,\phi \,+ \\
&\quad + \frac{1}{3}\, e^{2\phi}\,(\, G_{\mu\lambda\tau\sigma}\, G_{\nu}^{\;\lambda\tau\sigma} \,-\, \frac{1}{12}\, g_{\mu\nu}\, G_{\rho\lambda\tau\sigma}\, G^{\rho\lambda\tau\sigma}\,)\;, \\
\partial_\mu\Big(\sqrt{-g_{(8)}}\, P^{\mu ij}\Big) &= -\frac{2}{3}\,\nabla^2\,\phi\,\delta^{ij} \,+\, e^{2\phi}\, F^i_{\mu\nu}F^{\mu\nu\, j} \,+\, \frac{1}{36}\, e^{2\phi}\, G_{\mu\nu\tau\sigma}\, G^{\mu\nu\tau\sigma}\;\delta^{ij} \,+ \\
&\quad + \frac{1}{2}\, g^2\, e^{-2\phi}\,[\, T^{ik}\, T^{jk} \,-\, \frac{1}{2}\, T\, T^{ij} \,-\, \frac{1}{2}\,\delta^{ij}\,(\, T^{kl}T^{kl} \,-\, \frac{1}{2}\, T^2\,)\,]\;, \\
\partial_\mu\Big(\sqrt{-g_{(8)}}\, e^{2\phi}\, F^{\mu\nu\, i}\Big) &= -e^{2\phi}\, P_\mu^{ij}\, F^{\mu\nu\, j} \,-\, g\, g^{\mu\nu}\,\epsilon_{ijk}\, P_\mu^{jl}\, T^{kl}\;, \\
\partial_\mu\Big(\sqrt{-g_{(8)}}\;\, e^{2\phi}\, G^{\mu\nu\tau\sigma}\,\Big) &= 0\;.
\end{aligned} \tag{A.4}$$

Given a solution of the equations (A.4) one can write a solution of the D=11 theory by reverting the Salam-Sezgin reduction ansatz. For the eleven dimensional metric, the corresponding uplifting formula is:

$$ds^2_{11} \,=\, e^{-\frac{2}{3}\,\phi}\, ds^2_8 \,+\, 4\, e^{\frac{4}{3}\phi}\,(\, A^i \,+\, \frac{1}{2}\, L^i\,)^2\;, \tag{A.5}$$

where L^i is defined as:

$$L^i \,=\, \frac{2}{g}\,\tilde{w}^\alpha\, L^i_\alpha\;, \tag{A.6}$$

with $\tilde{w}^i$ being left invariant forms on the $SU(2)$ group manifold and g is the $SU(2)$ gauge coupling constant. The relation between the eleven and eight dimensional four-forms has been given in eq.(2.39).

B Effective lagrangian with 4-form

In this appendix we explain how to find effective lagrangians for a given ansatz for the eight dimensional fields when the four-form G is non-zero. Let us imagine that we substitute our ansatz for the metric and gauge field A^i_μ in the Salam-Sezgin lagrangian (A.3) and let us denote by f_i the different functions $f, h, \alpha, \cdots$ of the ansatz (including the dilaton and other scalar fields). As the four-form field has a radial component, we can represent it as B', where B is a potential and the prime denotes radial derivative. After integrating by parts to eliminate the second derivatives, the resulting lagrangian will be of the type:

$$\mathcal{L} \,=\, \tilde{\mathcal{L}}(\, f_i, f_i'\,) \,+\, a(\, f_i\,)\,(\, B'\,)^2\;, \tag{B.1}$$

where $a(\, f_i\,)$ does not depend on the derivatives of the f_i's. The equations of motion for $\mathcal{L}$ are:

$$\begin{aligned} \frac{d}{dr}\frac{\partial \tilde{\mathcal{L}}}{\partial f_i'} &= \frac{\partial \tilde{\mathcal{L}}}{\partial f_i} + (\, B'\,)^2\, \frac{\partial a}{\partial f_i}\,, \\ \frac{d}{dr}\Big[\, a\, B'\,\Big] &= 0\,, \end{aligned} \tag{B.2}$$

which, together with the corresponding zero energy condition, are equivalent to (A.4). Integrating the equation for B we get:

$$B' = \frac{\Lambda}{a(\, f_i\,)}\,, \tag{B.3}$$

where Λ is a constant. This is precisely our ansatz for G in eqs.(2.3) and (3.3). Substituting the value of B' given in eq.(B.3) in the equation for the f_i's, one gets:

$$\frac{d}{dr}\frac{\partial \tilde{\mathcal{L}}}{\partial f_i'} = \frac{\partial \tilde{\mathcal{L}}}{\partial f_i} + \frac{\Lambda^2}{a^2}\frac{\partial a}{\partial f_i} = \frac{\partial}{\partial f_i}\Big(\, \tilde{\mathcal{L}} - \frac{\Lambda^2}{a}\,\Big)\,, \tag{B.4}$$

and, therefore, the effective lagrangian for the f_i's is:

$$\mathcal{L}_{eff} = \tilde{\mathcal{L}}(\, f_i, f_i'\,) - \frac{\Lambda^2}{a(\, f_i\,)}\,. \tag{B.5}$$

Indeed, the Euler-Lagrange equations for $\mathcal{L}_{eff}$ are precisely (B.4). Notice the change of sign in the last term of $\mathcal{L}_{eff}$ as compared with the corresponding one in $\mathcal{L}$. This sign flip has been taken into account is eqs.(2.14) and (3.7) and is crucial to find the superpotentials. Equivalently, one can obtain $\mathcal{L}_{eff}$ by eliminating the cyclic coordinate B by constructing the Routhian $\mathcal{R}$ as:

$$\mathcal{R} = \mathcal{L} - B'\,\frac{\partial \mathcal{L}}{\partial B'}\,. \tag{B.6}$$

Clearly $\mathcal{R} = \mathcal{L}_{eff}$.

C General dependence of the metric on the 4-form

In sections 2 and 3 we have concluded that the effect of the four-form on the metric, as compared to the one obtained when the four-form is set to zero, is just the introduction of some warp factors. In this appendix we will prove that the validity of this result goes beyond the particular cases studied in the main text and, under some conditions, it holds in general.

Let us suppose that we adopt the following ansatz for the eight-dimensional metric and 4-form:

$$\begin{aligned} ds_8^2 &= e^{2f}\, dx_{1,2}^2 + \sum_{i=1}^{4} e^{2h_i}\,(\, E^i\,)^2 + dr^2\,, \\ G_{\underline{x^0x^1x^2r}} &= \Lambda\, e^{-\sum h_i - 2\phi} \equiv \Lambda\, e^{-\phi}\,\xi(\,\phi, h_i\,)\,, \end{aligned} \tag{C.1}$$

where E^i are some vierbiens, which we will assume to be independent of the radial coordinate r, Λ is a constant and we have defined the function $\xi(\,\phi, h_i\,)$. The equation of motion for G is satisfied when G has the form given in eq.(C.1). All the dependence on r is included in the functions f, h_i and ϕ. We will assume that we have also some scalar fields λ_i. These functions satisfy certain first-order BPS equations of the type:

$$\begin{aligned}
\frac{d}{dr} f &= \Gamma_f\,(\,\phi, h_i, \lambda_i) + \frac{\Lambda}{2}\,\xi(\,\phi, h_i\,)\,, \\
\frac{d}{dr} h_i &= \Gamma_{h_i}\,(\,\phi, h_i, \lambda_i) - \frac{\Lambda}{2}\,\xi(\,\phi, h_i\,)\,, \\
\frac{d}{dr} \phi &= \Gamma_\phi\,(\,\phi, h_i, \lambda_i) - \frac{\Lambda}{2}\,\xi(\,\phi, h_i\,)\,, \\
\frac{d}{dr} \lambda_i &= \Gamma_{\lambda_i}\,(\,\phi, h_i, \lambda_i)\,,
\end{aligned} \tag{C.2}$$

where the functions Γ of the right-hand side depend on the particular case we are considering. The only property we will need of these functions is that they satisfy the following homogeneity condition:

$$\Gamma(\,\phi + \gamma\,,\, h_i + \gamma\,, \lambda_i\,) = e^{-\gamma}\,\Gamma(\,\phi\,,\, h_i\,, \lambda_i\,)\,, \tag{C.3}$$

where γ is an arbitrary function. In all the cases studied here and in refs. [14, 16] the Γ's satisfy (C.3). On the other hand, from the definition of $\xi(\,\phi\,,\, h_i\,)$ one has:

$$\xi(\,\phi + \gamma\,,\, h_i + \gamma\,) = e^{-5\gamma}\,\xi(\,\phi\,,\, h_i\,)\,. \tag{C.4}$$

Let us now consider a function η such that solves the following differential equation:

$$\frac{d\eta}{dr} = -\frac{\Lambda}{2}\,\xi(\,\phi, h_i\,)\,, \tag{C.5}$$

and let us define the functions:

$$\tilde{f} = f + \eta\,, \qquad \tilde{h}_i = h_i - \eta\,, \qquad \tilde{\phi} = \phi - \eta\,. \tag{C.6}$$

If we now introduce a new radial variable $\tilde{r}$ such that:

$$\frac{dr}{d\tilde{r}} = e^{\eta}\,, \tag{C.7}$$

then, it is straightforward to prove that $\tilde{f}$, $\tilde{h}_i$ and $\tilde{\phi}$ and λ satisfy the following differential equations:

$$\begin{aligned}
\frac{d}{d\tilde{r}} \tilde{f} &= \Gamma_f\,(\,\tilde{\phi}, \tilde{h}_i, \lambda_i)\,, \\
\frac{d}{d\tilde{r}} \tilde{h}_i &= \Gamma_{h_i}\,(\,\tilde{\phi}, \tilde{h}_i, \lambda_i)\,,
\end{aligned}$$

$$
\begin{aligned}
\frac{d}{d\tilde r}\,\tilde\phi \;&=\; \Gamma_\phi\,(\,\tilde\phi, \tilde h_i, \lambda_i)\;,\\
\frac{d}{d\tilde r}\,\lambda_i \;&=\; \Gamma_{\lambda_i}\,(\,\tilde\phi, \tilde h_i, \lambda_i)\;,
\end{aligned}
\tag{C.8}
$$

which are the same as in those for the same system without the 4-form. Moreover, if we define the function H as:

$$
H\,\equiv\,e^{4\eta}\;,
\tag{C.9}
$$

then, the uplifted metric is:

$$
\begin{aligned}
ds^2_{11} \;&=\; H^{-\frac{2}{3}}\,e^{2\tilde f\,-\,\frac{2}{3}\,\tilde\phi}\,dx^2_{1,2}\,+\\
&+H^{\frac{1}{3}}\,\Big[\,\sum_i\,e^{2\tilde h_i\,-\,\frac{2}{3}\tilde\phi}(\,E^i\,)^2\,+\,e^{-\frac{2}{3}\tilde\phi}\,d\tilde r^2\,+\,4\,e^{\frac{4}{3}\tilde\phi}\,\Big(\,A_i\,+\,\frac{1}{2}\,L_i\,\Big)^2\,\Big]\;.
\end{aligned}
\tag{C.10}
$$

It is clear from eq. (C.10) that the effect of the 4-form on the metric is the introduction of some powers of H which distinguish among the directions parallel and orthogonal to the form. Moreover, it is easy to verify from the equation satisfied by η that the harmonic function H satisfies:

$$
\frac{dH}{d\tilde r}\,=\,-2\Lambda\;\xi(\,\tilde\phi\,,\,\tilde h^i\,)\,=\,-2\Lambda e^{-\sum\tilde h_i\,-\,\tilde\phi}\quad,
\tag{C.11}
$$

and, thus, if we know the solution without form, we can integrate the right-hand side of the last equation and find the expression of H. Notice that when $\Lambda = 0$ we can take $H =$ constant. In this case the components of the metric parallel to the 4-form are constant provided that $\tilde\phi = 3\tilde f$ solves eq. (C.8), which can only happen if $\Gamma_\phi = 3\Gamma_f$. This condition holds for all the systems studied here and in refs. [14, 16]. Moreover, if $\tilde\phi = 3\tilde f$ one can verify that the uplifted 4-form is such that $F_{x^0x^1x^2\tilde r} = \partial_{\tilde r}\,(\,H^{-1}\,)$.

As a illustration of the general formalism we have developed above, let us consider the case of a flat D6-brane with flux. In this situation there are no scalar fields λ excited and the ansatz for the metric is [7]:

$$
ds^2_8\,=\,e^{2f}\,dx^2_{1,2}\,+\,e^{2h}\,dy^2_4\,+\,dr^2\;.
\tag{C.12}
$$

The functions Γ appearing in the first-order system (C.2) are $\Gamma_f = \Gamma_h = \frac{\Gamma_\phi}{3} = \frac{g}{8}\,e^{-\phi}$. If we change to a new variable t such that $d\tilde r\,=\,e^{-\tilde\phi}\,dt$, we can write the solution of the system (C.8) as $\tilde f\,=\,\tilde h\,=\,\frac{\tilde\phi}{3}\,=\,\frac{g}{8}\,t$. Moreover, for the case at hand $\xi(\,\tilde\phi\,,\,\tilde h\,)\;=\;e^{-4\tilde h-\tilde\phi}$ and, by plugging this result in eq. (C.11) and taking $g = 1$, we get that the harmonic function is:

$$
H\,=\,-2\Lambda\int e^{-4\tilde h-\tilde\phi}d\tilde r\,=\,-2\Lambda\int e^{-4\tilde h}dt\,=\,1\,+\,4\Lambda\,e^{-\frac{t}{2}}\;,
\tag{C.13}
$$

where we have fixed the integration constant to recover the solution with $\Lambda = 0$ at $t \to \infty$. The eleven dimensional metric is readily obtained from the uplifting formula (A.5). Since there are no SU(2) gauge fields excited in this flat case [7], we get:

$$
ds^2_{11}\,=\,H^{-\frac{2}{3}}\,dx^2_{1,2}\,+\,H^{\frac{1}{3}}\,\Big(\,dy^2_4\,+\,e^{\frac{t}{2}}\,(dt^2\,+\,16\,d\Omega^2_3)\Big)\;.
\tag{C.14}
$$

Introducing a new variable ρ as $\rho = \frac{4}{\sqrt{N}}\, e^{\frac{t}{4}}$, the metric (C.14) can be put in the form:

$$ds^2_{11} = \Big[H(\rho) \Big]^{-\frac{2}{3}} dx^2_{1,2} + \Big[H(\rho) \Big]^{\frac{1}{3}} \Big(dy_4^2 + N(d\rho^2 + \rho^2\, d\Omega_3^2) \Big) , \tag{C.15}$$

where $H(\rho)$ is given by:

$$H(\rho) = 1 + \frac{64\Lambda}{N}\,\frac{1}{\rho^2} . \tag{C.16}$$

Notice that, as pointed out in the main text, the harmonic function of the D2-brane $H(\rho)$ appearing in the metric (C.15) is not in its near horizon limit. Actually, if one drops the 1 on the right-hand side of eq. (C.16), one can check that (C.14) coincides with the metric of the standard near horizon D2-D6 intersection.

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